Arc length

In this lesson we'll look at how to find the length of an arc when we're given the radius and the measure of a central angle in degrees.

Arc length

An arc can be measured in degrees, but it can also be measured in units of length. An arc measured in degrees is called the measure of the arc, and an arc being measured in units of length is called the arc length.

Recall that the circumference of a circle is the distance, or total length, around the circle. An arc is part of a circle's circumference, so it makes sense to use the circumference of a circle as part of the arc length formula. The formula for the circumference of a circle is $C = 2\pi r$, where r is the radius of the circle. The measure of a circle's full circumference is $360°$ degrees. When we find arc length, we use

$$L = \frac{m}{360} \cdot 2\pi r$$

where L is the arc length, m is the measure of the arc in degrees, and r is the radius of the circle.

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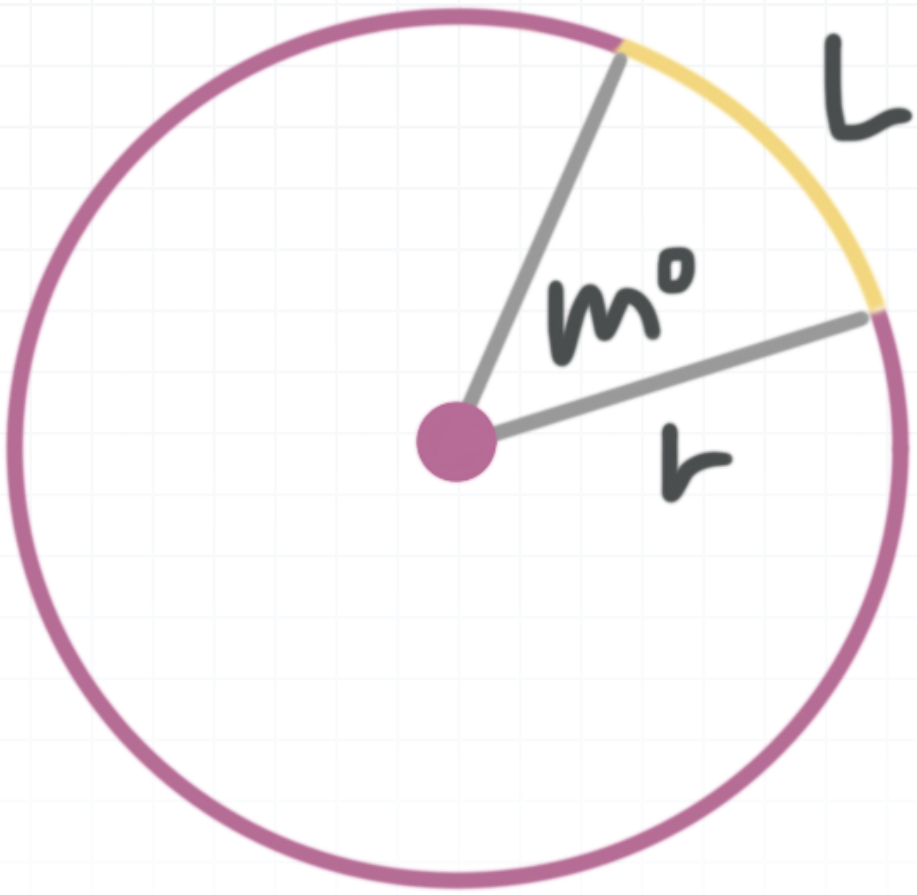

Many textbooks will refer to the arc length with the lowercase letter s. Let's start by working through an example.

Example

The length of the radius of circle Q is 12 cm and the measure of angle MQN is $86°$. What is the arc length of $\overset{\frown}{MN}$?

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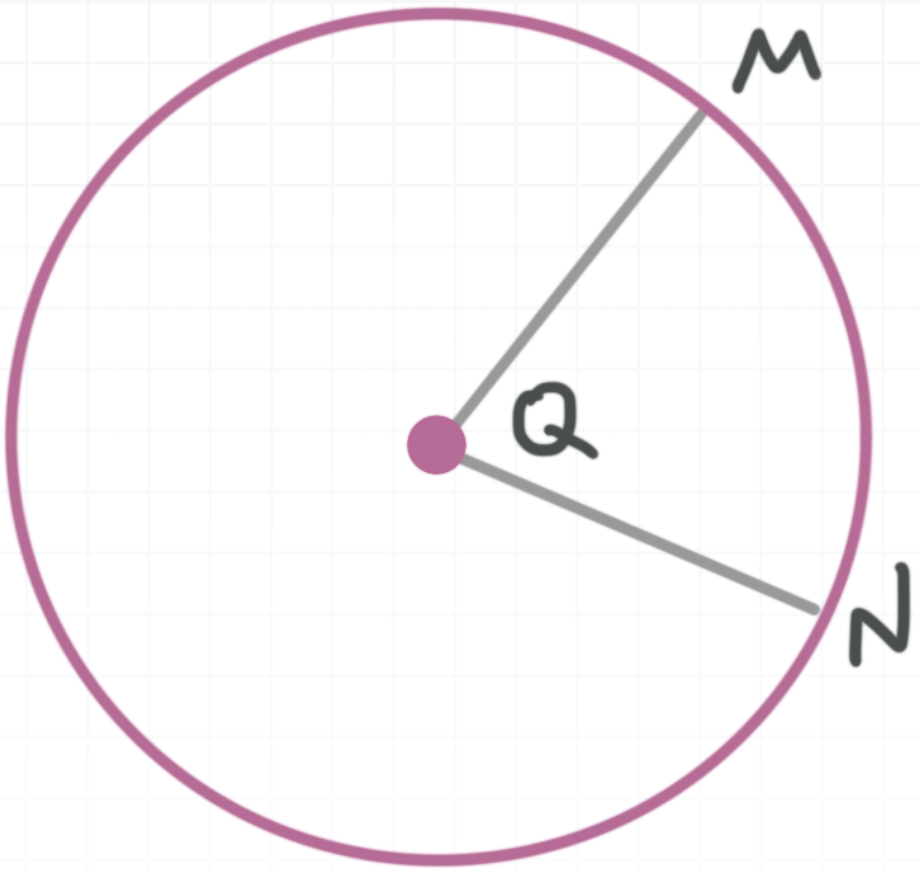

We know the radius of circle Q is 12 cm and the central angle of $\overset{\frown}{MN}$ measures $86°$, so $m\overset{\frown}{MN} = 86°$. Now we can use the arc length formula.

$$L = \frac{m}{360} \cdot 2\pi r$$

$$L = \frac{86}{360} \cdot 2\pi(12) \text{ cm}$$

$$L = \frac{86}{360} \cdot 2\pi(12) \text{ cm}$$

$$L = \frac{43}{180} \cdot 24\pi \text{ cm}$$

$$L = \frac{86}{15}\pi \text{ cm}$$

$$L \approx 18.01 \text{ cm}$$

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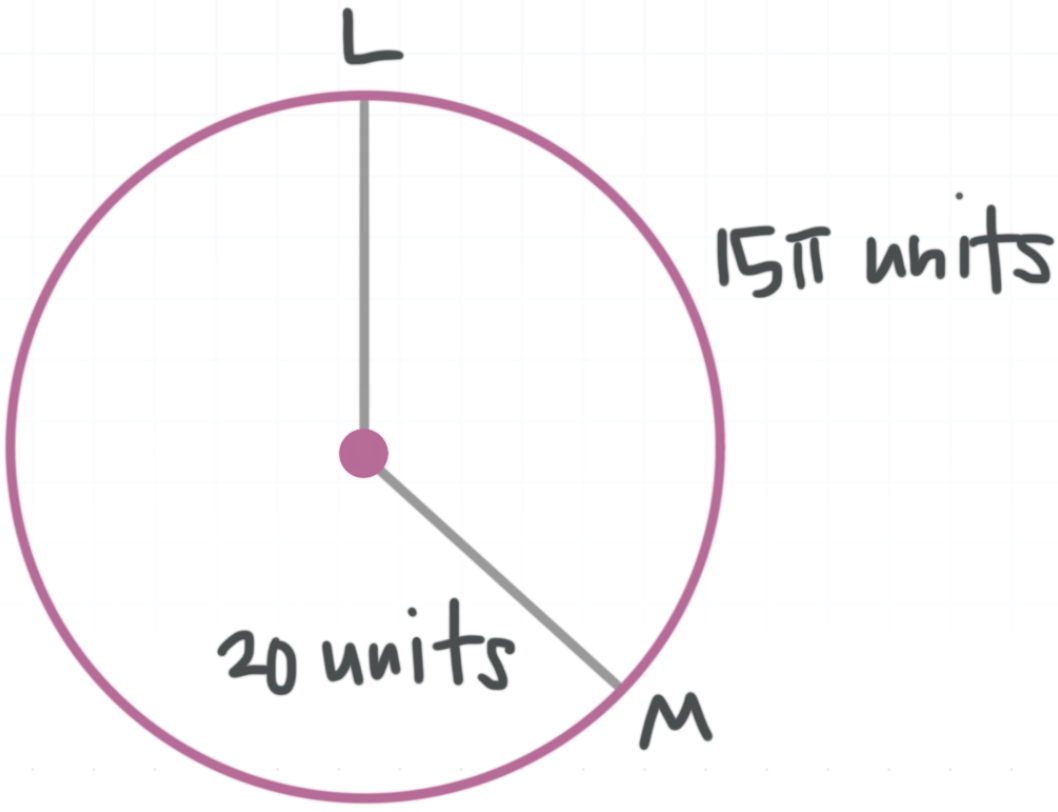
Sometimes you will be given the arc length and asked to find the measure of the arc or central angle.

Example

Find the measure of the central angle (in degrees) that belongs to arc LM.



Here we need to use the arc length formula to solve for the central angle. Remember that the measure of the arc is equal to the measure of the central angle that belongs to the arc.

$$L = \frac{m}{360} \cdot 2\pi r$$

We know $L = 15\pi$ units and $r = 20$ units, so we use the arc length formula to solve for the angle measure in degrees.

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$$L = \frac{m}{360} \cdot 2\pi r$$

$$15\pi = \frac{m}{360} \cdot 2\pi(20)$$

$$15\pi = \frac{m}{360} \cdot 40\pi$$

$$\frac{15\pi}{40\pi} = \frac{m}{360}$$

$$\frac{3}{8} = \frac{m}{360}$$

$$\frac{3}{8} \cdot 360 = m$$

$$m = 135^\circ$$

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